A Decomposition Approach for the Global Optimization of Nonlinear Network Design Problems

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Abstract. The optimal design of nonlinear networks yields a mixed-integer nonlinear programming (MINLP) problem with a nonconvex relaxation, whose computational complexity scales rapidly with the size of the network. This work presents a decomposition approach that decouples nonlinear calculations from the topological design problem, of combinatorial complexity. The design optimization problem is formulated as a mixed-integer linear model (MILP) that operates without embedded potential-loss constraints. Each candidate solution is subjected to an external feasibility evaluation, whose outcome feeds infeasibility cuts derived from minimal infeasible sectors. This mechanism guarantees global optimality independently of the nonlinear correlation employed. The approach is motivated by the design of multiphase gathering networks in unconventional oil and gas production.

Keywords: Nonlinear Network Design, MINLP, Decomposition, Infeasibility Cuts, Multiphase Gathering Networks.

Estrategia de Descomposición para la Optimización Global del Diseño de Redes con Restricciones No Lineales

Resumen. El diseño óptimo de redes con restricciones no lineales constituye un problema de programación mixta entera no lineal (MINLP) con relajación no convexa, cuya complejidad computacional escala rápidamente con el tamaño de la red. En este trabajo se propone un enfoque de descomposición que separa los cálculos de capacidad de los arcos del problema de diseño de la topología de la red, de naturaleza combinatoria. El problema de diseño se formula como un modelo mixto entero lineal (MILP) que opera sin restricciones de caída de potencial. Sobre cada solución candidata se realiza una evaluación de factibilidad externa, cuyo resultado alimenta cortes enteros derivados del reconocimiento de sectores infactibles. El procedimiento garantiza la optimalidad global de las soluciones, independientemente de la correlación no lineal empleada. El enfoque está motivado por el diseño de redes de recolección de flujos multifásicos en la producción de petróleo y gas no convencional.

Palabras clave: Diseño de Redes, MINLP, Descomposición, Cortes Enteros, Redes de Recolección con Flujo Multifásico.

1 Introduction

Nonlinear network design refers to the class of problems in which flows are driven by a potential, and the potential loss on each arc follows a nonlinear function with the flow rate (Raghunathan, 2013). Applications arise in electrical circuits, water distribution systems, structural networks, and, more recently, in the gathering of oil and gas production from unconventional formations (Drouven et al., 2023; Cafaro et al., 2022). In all cases, the objective is to select appropriate resistances (or, equivalently, pipeline diameters) at minimum total cost, such that potential (or pressure) limits are satisfied at the demand nodes. The discrete nature of resistance choices, combined with the nonlinearity of the potential-loss constraints, places the problem in the class of an MINLP, with nonconvex relaxation.

The computational difficulty is well established. Embedding nonlinear correlations directly into the optimization model introduces highly nonconvex constraints, making global optimization intractable as the number of nodes in the network grows. In the context of water distribution networks, Bragalli et al. (2012) propose an MINLP approach with a continuous reformulation of the cost function and a branch-and-bound algorithm, but their method is only guaranteed to find locally optimal solutions. D'Ambrosio et al. (2015) survey mathematical programming techniques for water network optimization, highlighting the persistent challenge of nonconvexity in potential-loss constraints. Raghunathan (2013) assumes a simplified version of the problem in which potential-loss functions are convex, thus yielding convex NLP relaxations of the MINLP, finally certifying the global optimality of solutions through a linearization-based LP/NLP branch-and-cut algorithm. Nevertheless, its effectiveness significantly diminishes on larger instances. In the context of multiphase oil and gas gathering networks, Trucco et al. (2026) develop an MINLP formulation and a successive relaxation algorithm based on piecewise-constant underestimations of the multiphase pressure-drop correlations, shown to be effective for two-echelon (star-star) designs. Both approaches share a fundamental limitation: the solution strategies are tightly coupled to a specific form of potential-loss functions, and computational tractability deteriorates sharply when network topologies become more complex.

This paper proposes a decomposition approach that addresses both limitations simultaneously. By removing nonlinear correlations entirely from the optimization model, the network design recovers a pure combinatorial, mixed-integer linear form. Feasibility is then evaluated externally on each candidate design, and infeasibility certificates are fed back as valid cuts. The approach guarantees global optimality independently of the nonlinear correlation employed. The main contributions of this work are: (i) a decomposition framework that decouples the potential-loss evaluation from the combinatorial search; (ii) the concept of Minimal Infeasible Trees (MITs) as a mechanism for generating strong, valid cuts; and (iii) a global optimality argument that holds under any potential-loss correlation, without convexity requirements.

2 Problem Formulation

Following Raghunathan (2013), we consider a directed graph $G = (N, E)$ where N is the set of nodes and E the set of arcs. Each node $i \in N$ is associated with a potential π_i, and each arc $(i,j) \in E$ carries a flow $I_{i,j}$. The potential loss on arc (i,j) with resistance r is given by a function $\varphi_r(I_{i,j}) \cdot l_{i,j}$, where φ_r is strictly increasing and $l_{i,j}$ denotes the arc length. A set of source nodes $N^{src} \in N$ is considered, each of them with a prescribed potential, and demand nodes must satisfy a minimum potential requirement π^{min}. The design objective is to select, for each arc, one resistance from a discrete set R, such that all potential requirements are met, with the aim of minimizing the total cost.

The specific instantiation motivating this work is the optimal design of gathering pipeline networks for unconventional oil and gas production (Trucco et al., 2026). Here, potential corresponds to fluid pressure, resistance choices correspond to standard commercial pipeline diameters, and the potential-loss function captures multiphase frictional pressure drops. The network must gather production from a set of wellpads and route flows to separation facilities over a multiperiod planning horizon, with the objective of minimizing the net present cost (NPC) of pipeline and facility installation.

Three features distinguish this setting from the general formulation of Raghunathan (2013) and merit explicit treatment:

Imposed tree topology. Flow splitting is not allowed; each node routes its production through exactly one downstream arc. The network is therefore a tree-like network, rooted at a separation facility. This structural constraint simplifies pressure propagation considerably: given a fixed topology and diameter selection, pressures can be computed by a simple procedure, with no need to solve a system of equations.

Fixed and limited supply. Unlike water distribution problems, where the goal is to satisfy a known demand at sink nodes, supply is fixed and determined by the production forecast of each wellpad over time. The optimization selects which sources to connect and how to size the network to transport their production at minimum cost, subject to hydraulic feasibility.

Minimum-potential rule. When multiple flows converge at a junction node, the resulting pressure equals the minimum outlet pressure among all incoming streams. Higher-pressure streams are throttled down via control valves to avoid backflow. This rule is both physically motivated and operationally imposed: pipeline diameters are installed once, and cannot be resized as wellhead pressures and fluid compositions evolve over the planning horizon. A diameter selection must therefore be feasible across all time periods, and a Minimal Infeasible Tree identified at any single period constitutes a valid cut for the multiperiod problem.

3 Proposed Methodology

The core idea of the proposed decomposition approach is to remove hydraulic correlations entirely from the optimization model. The combinatorial problem is formulated as an MILP that determines the network topology, i.e. the connections between wellpads, and the diameter assigned to each arc, with the objective of minimizing the net present cost (NPC) of pipeline installations. Hydraulic constraints are absent from this model. Since pipeline costs increase with diameter, the model will naturally select the smallest feasible diameter for each arc in the absence of pressure-loss constraints. At the first iterations, the resulting solutions are typically infeasible, but interestingly, they represent lower bounds for the NPC.

Given a candidate design, hydraulic feasibility is checked by propagating pressures downstream along the tree from each wellpad to the separation facility. Under the imposed tree topology, this evaluation requires a single-path procedure, with no system of equations to solve. Most importantly, it is applicable to any hydraulic correlation, regardless of convexity or smoothness. If the delivery pressure at the separation facility meets the minimum requirement across all time periods, the design is feasible and, as argued below, globally optimal.

When a candidate design is infeasible, the source of infeasibility can be attributed to a particular sector of the network, rather than the configuration as a whole. The concept of Minimal Infeasible Tree (MIT) is introduced in this work as a subtree such that, if the same combination of arcs and diameters appears in any candidate solution, the overall network is guaranteed infeasible, regardless of topology and diameter choices elsewhere. MITs are identified by a recursive algorithm that traverses the tree upstream from the infeasible delivery node, tracing the pressure deficit to its origin and collecting the minimal set of arcs responsible for the infeasibility. A single infeasible candidate solution may contain multiple independent MITs, each of which generates a separate cut. This is the mechanism through which the approach avoids exhaustive enumeration: one hydraulic evaluation can simultaneously eliminate a large class of infeasible configurations.

Each MIT-derived cut is a valid infeasibility certificate: it excludes only configurations that prove to generate excessive pressure losses, and no feasible solution is ever pruned. Since the master MILP without hydraulic constraints is a relaxation of the full problem, its optimal value provides a lower bound on the true minimum NPC at every iteration. When the algorithm terminates with a feasible solution, all configurations with strictly lower NPC have been excluded by valid cuts, and the returned solution is therefore globally optimal. These cuts can be derived from any hydraulic feasibility check procedure, including simulation-based models and correlations. Furthermore, because the minimum-potential rule causing infeasibility at a single time period can be propagated as a cut over the full planning horizon, each MIT generated from one period is simultaneously valid for all periods, strengthening the pruning beyond what a single-period analysis would yield.

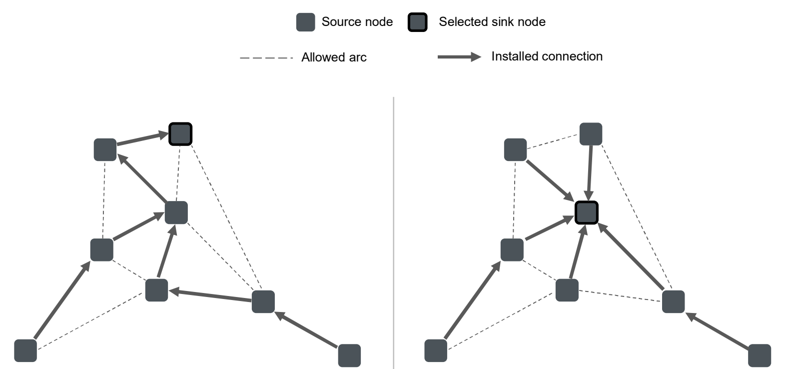

Fig. 1. Progress of the decomposition algorithm on an 8-node instance. First iteration: relaxed MILP solution with minimum diameters, hydraulically infeasible (left). Optimal solution found at iteration 164 (right).

4 Preliminary Results

To assess the computational performance of the proposed decomposition, a set of 100 randomly generated network instances have been solved to global optimality. Each instance comprises 8 source nodes, with three available pipeline diameter options and four candidate locations for the separation facility. Figure 1 illustrates the algorithm's progress on a representative instance: the initial relaxed MILP solution is shown in the left panel of Figure 1, which assigns minimum diameters throughout and is generically infeasible; at the right panel of Figure 1, the globally optimal design is depicted, which is returned at iteration 164, after successive MIT-derived cuts have eliminated infeasible, lower-cost configurations.

Across the 100 instances, the algorithm requires, on average, 350 seconds of CPU time and 210 iterations to reach the globally optimal solution. The per-iteration CPU time grows as the search progresses, since accumulated MIT-derived cuts increase the size of the master MILP. This behavior is expected and inherent to the subtractive enumeration mechanism. Scalability limitations become pronounced for larger instances: networks comprising 12 nodes require approximately two hours and 450 iterations on average, while no optimal solution has been obtained for 16-node instances within a 10-hour time limit. Conversely, the method performs very efficiently on small networks, solving 4-node instances in less than one second (0.54 s) and 7-node instances in less than one minute (32.6 s). These results confirm that the approach is well suited for small- to medium-scale problems and motivate the development of partition-based strategies to extend its reach to larger networks.

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